Calculus Warm Up #5-2

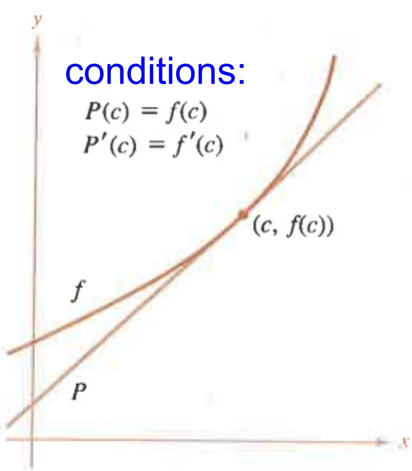
Test for convergence. State the test(s) used and give a complete conclusion.

1) $\sum_{n=1}^{\infty} \frac{(-1)^{n+1} 5}{n}$

2) $\sum_{n=1}^{\infty} \frac{n}{2n^2+1}$

10.7 Taylor Polynomials & Approximations

Using a polynomial, P(x), to approximate another function, f(x), near a common point, (c, f(c)).



The line P(x) can be used to approximate outcomes on f(x) that are near x = c.

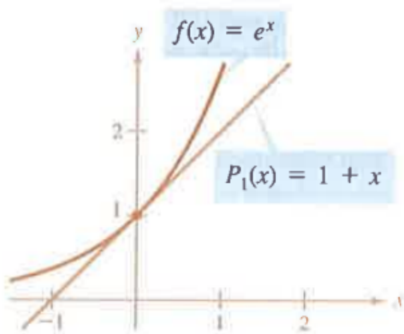
Ex: Find a first degree polynomial, $P(x)$, to approximate $f(x) = e^x$ near $x = 0$.

$f'(x) = e^x$

<u>conditions:</u>

$P(c) = f(c)$ $\quad P(0) = f(0) = e^0 = 1$ (y-int)

$P'(c) = f'(c)$ $\quad P'(0) = f'(0) = 1$ (slope)



Approximations are not very good as we move away from c = 0

Second degree polynomial, $P(x)$, to approximate $f(x) = e^x$ near $x = 0$.

$f'(x) = e^x$ $\quad f''(0) = 1$

$f''(x) = e^x$

<u>conditions:</u>

$P(c) = f(c)$

$P'(c) = f'(c)$ $\quad P_1(x) = x + 1$

$P''(c) = f''(c)$ $\quad P_2(x) = \int P_1(x)\,dx$

$$P_2(x) = \frac{x^2}{2} + x + C$$

plug in $(0,1) \rightarrow$ to get $C = 1$

so $P_2(x) = \frac{1}{2}x^2 + x + 1$

Second degree polynomial, P(x), to approximate $f(x) = e^x$ near x = 0.

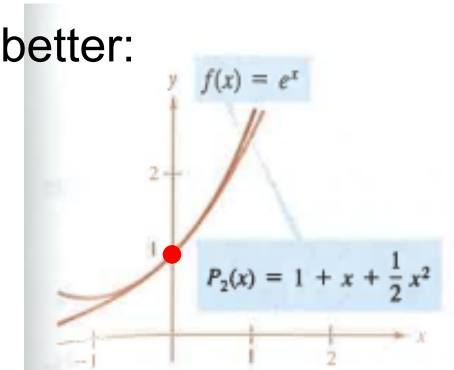
<u>conditions:</u>

$P(c) = f(c)$

$P'(c) = f'(c)$

$P''(c) = f''(c)$

better:



We can continue to make further restrictions, finding n derivatives...

$$P_n(x) = 1 + x + \frac{1}{2}x^2 + \frac{1}{3!}x^3 + \frac{1}{4!}x^4 + \cdot\cdot + \frac{1}{n!}x^n$$

$$P_n(x) \approx f(x)$$

$$P_n(x) \approx e^x$$

In this example we centered around the common point where c = 0.

We can generalize for c = any x

A Taylor Polynomial centered at any $x = c$,

where $P(c) = f(c)$ through $P_n^{(n)}(c) = f^{(n)}(c)$

The nth Taylor Polynomial

$$P_n(x) = f(c) + f'(c)(x-c) + \frac{f''(c)(x-c)^2}{2!} + \frac{f'''(c)(x-c)^3}{3!} + \dots + \frac{f^{(n)}(c)(x-c)^n}{n!}$$

The nth Maclaurin Polynomial

centered at $c = 0$

$$P_n(x) = f(0) + f'(0)x + \frac{f''(0)x^2}{2!} + \frac{f'''(0)x^3}{3!} + \dots + \frac{f^{(n)}(0)x^n}{n!}$$

Ex: Find Taylor Polynomials for $f(x) = \ln x$

Centered at $c = 1$, P_0 through P_4

for $P_0 \rightarrow f(x) = \ln x \qquad f(1) = \ln 1 = 0$

$P_1 \rightarrow f'(x) = \frac{1}{x} \qquad f'(1) = 1$

$P_2 \rightarrow f''(x) = -\frac{1}{x^2} \qquad f''(1) = -1$

$P_3 \rightarrow f'''(x) = \frac{2}{x^3} \qquad f'''(1) = 2$

$P_4 \rightarrow f^{(4)}(x) = \frac{-6}{x^4} \qquad f^{(4)}(1) = -6$

$$P_1(x) = 0 + \frac{1(x-1)^1}{1!} = \boxed{x-1}$$

$$P_2(x) = P_1(x) + \frac{f''(1)(x-1)^2}{2!}$$

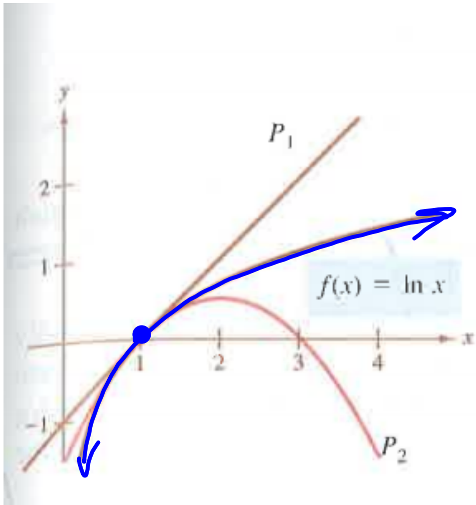
$$\boxed{P_2(x) = (x-1) - \frac{1}{2}(x-1)^2}$$

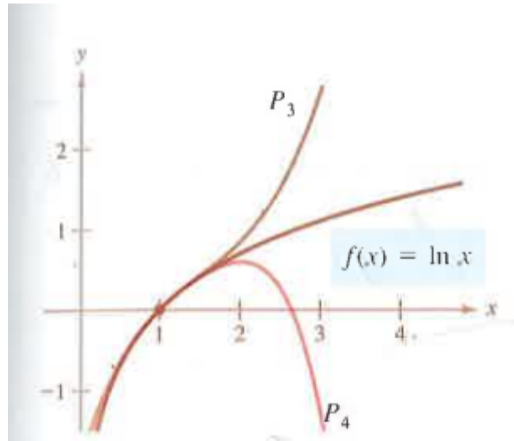
$$P_3(x) = (x-1) - \frac{1}{2}(x-1)^2 + \frac{f'''(1)(x-1)^3}{3!}$$

$$\boxed{P_4(x) = (x-1) - \frac{1}{2}(x-1)^2 + \frac{1}{3}(x-1)^3 - \frac{1}{4}(x-1)^4}$$

Ex: Find Taylor Polynomials for $f(x) = \ln x$

Centered at $c = 1$, P_0 through P_4





Ex: Find Maclaurin Polynomials for

$f(x) = \cos x$, P_0 , P_2 , P_4 , P_6 $\quad f(0) = 1$

$f'(x) = -\sin x \quad f'(0) = 0$

$f''(x) = -\cos x \quad f''(0) = -1$

$f'''(x) = \sin x \quad f'''(0) = 0$

$f^{(4)}(x) = \cos x \quad f^{(4)}(0) = 1$

$P_4(x) = 1 - \frac{x^2}{2} + \frac{x^4}{4!}$

$\underline{P_6(x) = 1 - \frac{x^2}{2} + \frac{x^4}{4!} - \frac{x^6}{6!}}$

$P_0(x) = 1 \qquad P_2(x) = 1 - \frac{x^2}{2}$

$P_1(x) = 1 + \frac{0x}{1!} \qquad P_3(x) = 1 - \frac{x^2}{2} + 0(\quad)$

Use P_6 to approximate $f(0.1)$.

$f(0.1) \approx P_6(0.1) \approx 0.995004165$

accurate to 9 dec. places.

HW: p. 615, # 1 - 19 odd,
(skip 9)

Classwork: BC MC - A Practice test

*Things we still need to learn:

#7, 10, 17, 26 (#13: derivatives of parametric equations. Try it!)
23

Red O, posted Mon.
Green O, posted Tues.

check answers:

1. D	6. B	11. B	16. E	21. E	26. C
2. C	7. B	12. D	17. D	22. A	27. E
3. C	8. D	13. B	18. B	23. B	28. A
4. E	9. A	14. C	19. D	24. D	
5. D	10. D	15. E	20. E	25. E	

MC - A

9) Tangent line approximation

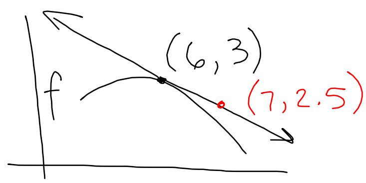
$y - 3 = -\frac{1}{2}(x-6)$

$y = -\frac{1}{2}(x-6) + 3 \rightarrow$ use to approx $f(7)$

$f(7) \approx -\frac{1}{2}(7-6) + 3$

$f(7) \approx 2.5$, since f is concave down

$f(7) < 2.5$ $\boxed{A}$



25) $\int_0^1 \frac{1}{x}\,dx$ does not exist.

Evaluate the improper integral:

$$\lim_{a \to 0} \int_a^1 \frac{1}{x}\,dx$$

$$= \lim_{a \to 0} \Big[\ln x\Big]_a^1$$

$$= \lim_{a \to 0} (\ln 1 - \ln a)$$

$$\ln 1 - \underbrace{\ln 0}_{\text{undefined.}}$$

28) $-1 \le \sin\left(\frac{x+1}{x^2}\right) \le 1$ hole @ $x=0$

horiz @ $y=0$

$$\lim_{x \to \pm\infty} \left[\sin\left(\frac{x+1}{x^2}\right)\right] = 0$$

bottom grows faster

So as $x \to \infty$

$\left(\frac{x+1}{x^2}\right) \to 0$

$\boxed{A}$

$\sin(0) = 0$

MC part B answers:

76. B
77. B
78. D
79. E
80. B
81. A
82. C
83. D
84. C
85. B
86. E
87. B
88. C
89. B
90. D
91. B
92. C

* Still need to learn:

#78, 86, 87, 90